The importance of resolution

The inference rule

```
A ∨ B ∨ C ∨ ... ∨ Z
D ∨ E ∨ F ∨ ... ∨ ¬Z
--------------------------
A ∨ B ∨ C ∨ D ∨ E ∨ F ∨ ...
```

Part of the original Prolog example:

```
eats(X, Y) :- cat(X), mouse(Y).
chases(X, Y) :- eats(X, Y).
chases(X, Y) :- dog(X), cat(Y).
cat(tom).
mouse(jerry).
```

Those are all reversed implications, e.g.
if `cat(X)` and `mouse(Y)` then `eats(X, Y)`

So what we've got is five things that are all true

```
(cat(X) ∧ mouse(Y) ⇒ eats(X, Y)) ∧
(eats(X, Y) ⇒ chases(X, Y)) ∧
(dog(X) ∧ cat(Y) ⇒ chases(X, Y)) ∧
(cat(tom)) ∧
(mouse(jerry))
```

Propositional logic can't handle those sorts of formulae, so simplify:

```
(cx ∧ my ⇒ exy) ∧
(exy ⇒ chxy) ∧
(dx ∧ cy ⇒ chxy) ∧
(cx) ∧
(my)
```

And recall these equivalences:

$$A \Rightarrow Z \equiv Z \lor \neg A$$

so

$$A \land B \land C \Rightarrow Z \equiv Z \lor \neg(A \land B \land C) \equiv Z \lor \neg A \lor \neg B \lor \neg C$$

Our rule base is therefore

```
(exy ∨ ¬cx ∨ ¬my) ∧
(chxy ∨ ¬exy) ∧
(chxy ∨ ¬dx ∨ ¬cy) ∧
(cx) ∧
(my)
```

All nicely in Conjunctive Normal Form (CNF)

```
(exy ∨ ¬cx ∨ ¬my) ∧
(chxy ∨ ¬exy) ∧
(chxy ∨ ¬dx ∨ ¬cy) ∧
(cx) ∧
(my)
```

Suppose we want to know whether chxy or not.

> Resolve cx with exy ∨ ¬cx ∨ ¬my giving exy ∨ ¬my
>
> Resolve that with chxy ∨ ¬exy giving ¬my ∨ chxy
>
> Resolve that with my giving chxy
>
> Q.E.D.

The absolute technical truth.

> Logic programming actually works through proof by contradiction, attempt to prove the negation of what you want, and hope to fail.
>
> We wanted to prove chxy, so we and ¬chxy with our rule base:

```
(exy ∨ ¬cx ∨ ¬my) ∧
(chxy ∨ ¬exy) ∧
(chxy ∨ ¬dx ∨ ¬cy) ∧
(cx) ∧
(my) ∧
(¬chxy)
```

> and let resolution proceed exactly as above.
> At the end, when we were left with chxy, there is now one more step.
>
> Resolve that with ¬chxy, resulting in nothing at all, which is failure.
>
> That means that (our knowledge base) ∧ (¬chxy) is false.
>
> Our knowledge base is the definition of truth in this program, so it must be ¬chxy that is false, so according to our knowledge base, chxy is true.
>
> The only trick is knowing which two facts to apply resolution to at each step.