

a)

Modus Ponens: if you know that A implies B and you know that A is true, then you can deduce that B is true.

Knowing that if it is a Monday then we have class
and also knowing that it is a Monday,
I deduce that we have class.

b)

Resolution: Given any two clauses that are disjunctions (ors) or literals (variables or their nots), if any variable appears as a positive in one clause and a negative in the other, Then a third clause consisting of the two combined but with the shared variable completely removed can be deduced.
example: $(A \vee \neg B \vee C) \wedge (B \vee \neg D \vee F)$ gives $(A \vee C \vee \neg D \vee F)$

$$1: \quad A \vee \neg B \vee C \vee D$$

$$2: \quad C \vee \neg D \vee E \vee \neg F$$

$$3: \quad B \vee F \vee G \vee \neg H$$

$$4: \quad A \vee C \vee G$$

$$1 \text{ and } 2 \text{ give } 5: \quad A \vee \neg B \vee C \vee E \vee \neg F$$

$$1 \text{ and } 3 \text{ give } 6: \quad A \vee C \vee D \vee F \vee G \vee \neg H$$

$$2 \text{ and } 3 \text{ give } 7: \quad B \vee C \vee \neg D \vee E \vee G \vee \neg H$$

and so on. (e.g. 5 and 3 can resolve too)

c)

From the truth table, or remembering the rule, or just thinking about it, we get that $A \Leftarrow B$ is the same as $A \vee \neg B$.

We have $\neg C \vee \neg D \vee E \vee \neg F$, the only positive is E, so E must correspond with A in the desired implication, that means all the rest, $\neg C \vee \neg D \vee \neg F$ must correspond with $\neg B$.

$$\neg B \equiv \neg C \vee \neg D \vee \neg F, \text{ therefore}$$

$$B \equiv \neg(\neg C \vee \neg D \vee \neg F), \text{ therefore by MeMorgan's rule}$$

$$B \equiv C \wedge D \wedge F$$

so the final implication is $E \Leftarrow C \wedge D \wedge F$.

d)

on the next page.

You were given

```
num(z).
num(s(X)) :- num(X).

pred(s(X), X).

add(z, Y, Y).
add(s(X), Y, s(Z)) :- add(X, Y, Z).
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The solutions are

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sub(X, z, X).
sub(X, s(Y), Z) :- sub(X, Y, A), pred(A, Z).

mul(z, Y, z).
mul(s(X), Y, Z) :- mul(X, Y, A), add(A, Y, Z).

fac(z, s(z)).
fac(s(X), Y) :- fac(X, A), mul(A, s(X), Y).
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and I'm adding this to make it easily testable

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val(z, 0).
val(s(X), Y) :- val(X, Z), Y is Z + 1.
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Here is a session

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?- val(A, 6), val(B, 9), add(A, B, C), val(C, D).
A = s(s(s(s(s(s(z)))))),
B = s(s(s(s(s(s(s(s(s(z))))))))) ,
C = s(s(s(s(s(s(s(s(s(s(...))))))))) ,
D = 15 .

?- val(A, 14), val(B, 8), sub(A, B, C), val(C, D).
A = s(s(s(s(s(s(s(s(s(s(...))))))))) ,
B = s(s(s(s(s(s(s(s(z))))))) ,
C = s(s(s(s(s(s(z)))))) ,
D = 6 .

?- val(A, 6), val(B, 8), mul(A, B, C), val(C, D).
A = s(s(s(s(s(s(z)))))) ,
B = s(s(s(s(s(s(s(s(z))))))) ,
C = s(s(s(s(s(s(s(s(s(s(...))))))))) ,
D = 48 .

?- val(A, 6), fac(A, B), val(B, C).
A = s(s(s(s(s(s(z)))))) ,
B = s(s(s(s(s(s(s(s(s(s(...))))))))) ,
C = 720 .
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