

Some propositions

A: any particular day is a weekday (Mon-Fri)

B: any particular day is a weekend day (Sat and Sun)

C: any particular day is a Thursday

D: any particular day has a full moon

E: any particular day is in March

	A	B	C
Mon	1	0	0
Tue	1	0	0
Wed	1	0	0
Thur	1	0	1
Fri	1	0	0
Sat	0	1	0
Sun	0	1	0

$$P(A) = 5/7 \approx 0.714$$

$$P(B) = 2/7 \approx 0.286$$

$$P(C) = 1/7 \approx 0.143$$

The length of the Lunar cycle is 27.321661 days

$$P(D) = 1/27.321661 \approx 0.037$$

In every 400 year cycle:

There are 303 normal 365 day years

There are 97 leap 366 day years

$$303 \times 365 + 97 \times 366 = 146097 \text{ days}$$

Every year has one 31 day March

$$400 \times 31 = 12400$$

$$P(E) = 12400/146097 \approx 0.085$$

Column G: correct answer

Column F: calculated by normal formula:

$$P(X \wedge Y) = P(X) \times P(Y)$$

$$P(X \vee Y) = P(X) + P(Y) - P(X) \times P(Y)$$

	G	F	
$P(A \wedge B)$	0 = 0.000	10/49 $\approx$ 0.204	(weekday and weekend)
$P(A \vee B)$	1 = 1.000	39/49 $\approx$ 0.796	(weekday or weekend)
$P(A \wedge C)$	1/7 $\approx$ 0.143	5/49 $\approx$ 0.102	(weekday and Thursday)
$P(A \vee C)$	5/7 $\approx$ 0.714	42/49 $\approx$ 0.755	(weekday or Thursday)
$P(B \wedge C)$	0 = 0.000	2/49 $\approx$ 0.041	(weekend and Thursday)
$P(B \vee C)$	3/7 $\approx$ 0.429	19/49 $\approx$ 0.388	(weekend or Thursday)
$P(A \wedge D)$	$\approx$ 0.026	$\approx$ 0.026	(weekday and full moon)
$P(A \vee D)$	$\approx$ 0.725	$\approx$ 0.725	(weekday or full moon)
$P(B \wedge D)$	$\approx$ 0.010	$\approx$ 0.010	(weekend and full moon)
$P(B \vee D)$	$\approx$ 0.312	$\approx$ 0.312	(weekend or full moon)
$P(A \wedge E)$	$\approx$ 0.061	$\approx$ 0.061	(weekday and March)
$P(A \vee E)$	$\approx$ 0.738	$\approx$ 0.738	(weekday or March)
$P(D \wedge E)$	$\approx$ 0.003	$\approx$ 0.003	(full moon and March)
$P(D \vee E)$	$\approx$ 0.118	$\approx$ 0.118	(full moon or March)
$P(B \wedge E)$	$\approx$ 0.024	$\approx$ 0.024	(weekend and March)
$P(B \vee E)$	$\approx$ 0.347	$\approx$ 0.347	(weekend or March)

For conditional probabilities $P(X | Y) = P(X \wedge Y) / P(Y)$

Column FB: $P(X \wedge Y)$ calculated by formula

Column FG: the correct value for $P(X \wedge Y)$ is used

	G	FB	FG
$P(B A)$	(what are the chances that it's a weekend once we know that it's a weekday)		
	0	$0.204/0.714 \approx 0.286$	$0/.714 = 0$
$P(B E)$	(what are the chances that it's a weekend once we know that it's in March)		
	$2/7 \approx 0.286$	$0.024/0.085 \approx 0.286$	$0.024/0.085 \approx 0.286$

In the universe of numbers between 1 and 400:

$P(A: \text{it's even}) = 0.5$ of the universe, 200 numbers

$P(B: \text{it's between 100 and 149}) = 0.125$ of the universe, 50 numbers

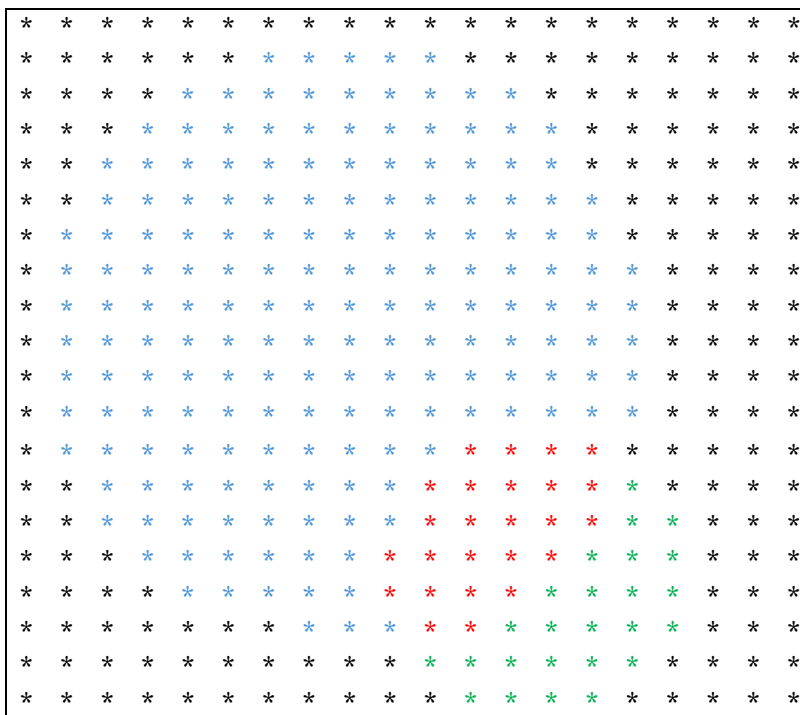
25 numbers are both, 0.0625 of the universe

175 items black: not even and not between 100 and 149

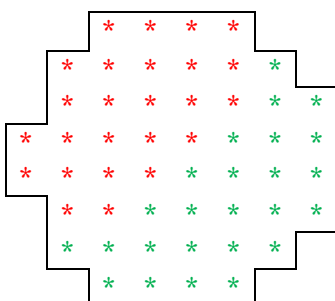
25 items are red: both even and between 100 and 149

175 items are blue: even but not between 100 and 149

25 items are green: between 100 and 149 but not even



In $P(A | B)$, the probability that A is true given that B is known to be true, we are given that B is true, so the world is reduced to those 50 items



And we are interested in the 25 red ones in which A is true (i.e. both A and B are true). They were 0.0625 of the original universe of 400 numbers, and so were the green ones. But both cover 0.5 of the universe we care about now.

So $P(A \mid B)$ and $P(\neg A \mid B)$ are both 0.5.

When our universe was reduced to the $P(\dots \mid B)$ area its size was multiplied by $P(B)$, 0.125, so to re-scale $P(A \wedge B)$ we must undo the original scaling down, divide by $P(B)$.