***2·14.** $\vdash . \sim(\sim p) \supset p$

Dem.

$\left[\text{Perm}\ \frac{\sim\{\sim(\sim p)\}}{q}\right] \vdash : p \vee \sim\{\sim(\sim p)\} . \supset . \sim\{\sim(\sim p)\} \vee p$ (1)

$[(1).*2\cdot13.*1\cdot11] \quad \vdash . \sim\{\sim(\sim p)\} \vee p$ (2)

$[(2).(*1\cdot01)] \quad \vdash . \sim(\sim p) \supset p$

***2·15.** $\vdash : \sim p \supset q . \supset . \sim q \supset p$

Dem.

$\left[*2\cdot05\ \frac{\sim p,\ \sim(\sim q)}{p,\quad r}\right] \vdash :. q \supset \sim(\sim q) . \supset : \sim p \supset q . \supset . \sim p \supset \sim(\sim q)$ (1)

$\left[*2\cdot12\ \frac{q}{p}\right] \quad \vdash . q \supset \sim(\sim q)$ (2)

$[(1).(2).*1\cdot11] \quad \vdash : \sim p \supset q . \supset . \sim p \supset \sim(\sim q)$ (3)

$\left[*2\cdot03\ \frac{\sim p,\ \sim q}{p,\quad q}\right] \vdash : \sim p \supset \sim(\sim q) . \supset . \sim q \supset \sim(\sim p)$ (4)

$\left[*2\cdot05\ \frac{\sim q,\ \sim(\sim p),\ p}{p,\quad q,\quad r}\right] \vdash :. \sim(\sim p) \supset p . \supset : \sim q \supset \sim(\sim p) . \supset . \sim q \supset p$ (5)

$[(5).*2\cdot14.*1\cdot11] \quad \vdash : \sim q \supset \sim(\sim p) . \supset . \sim q \supset p$ (6)

$\left[*2\cdot05\ \frac{\sim p \supset q,\ \sim p \supset \sim(\sim q),\ \sim q \supset \sim(\sim p)}{p,\quad q,\quad r}\right] \vdash ::$

$\sim p \supset \sim(\sim q) . \supset . \sim q \supset \sim(\sim p) : \supset :.$

$\sim p \supset q . \supset . \sim p \supset \sim(\sim q) : \supset : \sim p \supset q . \supset . \sim q \supset \sim(\sim p)$ (7)

$[(4).(7).*1\cdot11] \vdash :. \sim p \supset q . \supset . \sim p \supset \sim(\sim q) : \supset :$

$\sim p \supset q . \supset . \sim q \supset \sim(\sim p)$ (8)

$[(3).(8).*1\cdot11] \vdash : \sim p \supset q . \supset . \sim q \supset \sim(\sim p)$ (9)

$\left[*2\cdot05\ \frac{\sim p \supset q,\ \sim q \supset \sim(\sim p),\ \sim q \supset p}{p,\quad q,\quad r}\right] \vdash :: \sim q \supset \sim(\sim p) . \supset . \sim q \supset p :$

$\supset :. \sim p \supset q . \supset . \sim q \supset \sim(\sim p) : \supset : \sim p \supset q . \supset . \sim q \supset p$ (10)

$[(6).(10).*1\cdot11] \vdash :. \sim p \supset q . \supset . \sim q \supset \sim(\sim p) : \supset :$

$\sim p \supset q . \supset . \sim q \supset p$ (11)

$[(9).(11).*1\cdot11] \vdash : \sim p \supset q . \supset . \sim q \supset p$

Note on the proof of *2·15. In the above proof, it will be seen that (3), (4), (6) are respectively of the forms $p_1 \supset p_2$, $p_2 \supset p_3$, $p_3 \supset p_4$, where $p_1 \supset p_4$ is the proposition to be proved. From $p_1 \supset p_2$, $p_2 \supset p_3$, $p_3 \supset p_4$ the proposition $p_1 \supset p_4$ results by repeated applications of *2·05 or *2·06 (both of which are called "Syll"). It is tedious and unnecessary to repeat this process every time it is used; it will therefore be abbreviated into

"[Syll] $\vdash . (a) . (b) . (c) . \supset \vdash . (d)$,"

where (a) is of the form $p_1 \supset p_2$, (b) of the form $p_2 \supset p_3$, (c) of the form $p_3 \supset p_4$, and (d) of the form $p_1 \supset p_4$. The same abbreviation will be applied to a sorites of any length.

Also where we have "$\vdash . p_1$" and "$\vdash . p_1 \supset p_2$," and p_2 is the proposition to be proved, it is convenient to write simply

"$\vdash . p_1 . \supset$

$\vdash . p_2$,"

[etc.]

where "etc." will be a reference to the previous propositions in virtue of which the implication "$p_1 \supset p_2$" holds. This form embodies the use of *1·11 or *1·1, and makes many proofs at once shorter and easier to follow. It is used in the first two lines of the following proof.

***2·16.** $\vdash : p \supset q . \supset . \sim q \supset \sim p$

Dem.

$[*2\cdot12] \quad \vdash . q \supset \sim(\sim q) . \supset$

$[*2\cdot05] \quad \vdash : p \supset q . \supset . p \supset \sim(\sim q)$ (1)

$\left[*2\cdot03\ \frac{\sim q}{q}\right] \vdash : p \supset \sim(\sim q) . \supset . \sim q \supset \sim p$ (2)

$[\text{Syll}] \quad \vdash . (1) . (2) . \supset \vdash : p \supset q . \supset . \sim q \supset \sim p$

Note. The proposition to be proved will be called "Prop," and when a proof ends, like that of *2·16, by an implication between asserted propositions, of which the consequent is the proposition to be proved, we shall write "$\vdash$. etc. $\supset \vdash$. Prop". Thus "$\supset \vdash$. Prop" ends a proof, and more or less corresponds to "Q.E.D."

***2·17.** $\vdash : \sim q \supset \sim p . \supset . p \supset q$

Dem.

$\left[*2\cdot03\ \frac{\sim q,\ p}{p,\quad q}\right] \vdash : \sim q \supset \sim p . \supset . p \supset \sim(\sim q)$ (1)

$[*2\cdot14] \quad \vdash : \sim(\sim q) \supset q : \supset$

$[*2\cdot05] \quad \vdash : p \supset \sim(\sim q) . \supset . p \supset q$ (2)

$[\text{Syll}] \quad \vdash . (1) . (2) . \supset \vdash . \text{Prop}$

*2·15, *2·16 and *2·17 are forms of the principle of transposition, and will be all referred to as "Transp."

***2·18.** $\vdash : \sim p \supset p . \supset . p$

Dem.

$[*2\cdot12] \quad \vdash . p \supset \sim(\sim p) . \supset$

$[*2\cdot05] \quad \vdash . \sim p \supset p . \supset . \sim p \supset \sim(\sim p)$ (1)

$\left[*2\cdot01\ \frac{\sim p}{p}\right] \vdash : \sim p \supset \sim(\sim p) . \supset . \sim(\sim p)$ (2)

$[\text{Syll}] \quad \vdash . (1) . (2) . \supset \vdash : \sim p \supset p . \supset . \sim(\sim p)$ (3)

$[*2\cdot14] \quad \vdash . \sim(\sim p) \supset p$ (4)

$[\text{Syll}] \quad \vdash . (3) . (4) . \supset \vdash . \text{Prop}$

This is the complement of the principle of the *reductio ad absurdum*. It

*23·45. $\vdash : R \dot{\subset} S . R \dot{\subset} T . \supset . R \dot{\subset} S \dot{\cap} T$

*23·46. $\vdash : xRy . R \dot{\subset} S . \supset . xSy$

*23·47. $\vdash : R \dot{\subset} T . \supset . R \dot{\cap} S \dot{\subset} T$

*23·48. $\vdash : R \dot{\subset} S . \supset . R \dot{\cap} T \dot{\subset} S \dot{\cap} T$

*23·481. $\vdash : R = S . \supset . R \dot{\cap} T = S \dot{\cap} T$

*23·49. $\vdash : P \dot{\subset} Q . R \dot{\subset} S . \supset . P \dot{\cap} R \dot{\subset} Q \dot{\cap} S$

*23·5. $\vdash . R \dot{\cap} R = R$

*23·51. $\vdash . R \dot{\cap} S = S \dot{\cap} R$

*23·52. $\vdash . (R \dot{\cap} S) \dot{\cap} T = R \dot{\cap} (S \dot{\cap} T)$

*23·53. $R \dot{\cap} S \dot{\cap} T = (R \dot{\cap} S) \dot{\cap} T$ Df

*23·54. $\vdash :. R = S . \supset : R \dot{\subset} T . \equiv . S \dot{\subset} T$

*23·55. $\vdash :. R = S . \supset : T \dot{\subset} R . \equiv . T \dot{\subset} S$

*23·551. $\vdash : R = S . \supset . R \dot{\cup} T = S \dot{\cup} T$

*23·56. $\vdash . R \dot{\cup} R = R$

*23·57. $\vdash . R \dot{\cup} S = S \dot{\cup} R$

*23·58. $\vdash . R \dot{\subset} R \dot{\cup} S . S \dot{\subset} R \dot{\cup} S$

*23·59. $\vdash : R \dot{\subset} T . S \dot{\subset} T . \equiv . R \dot{\cup} S \dot{\subset} T$

*23·6. $\vdash :. x (R \dot{\cup} S) y . \equiv : R \dot{\subset} T . S \dot{\subset} T . \supset_T . xTy$

*23·61. $\vdash : R \dot{\subset} S . \supset . R \dot{\subset} S \dot{\cup} T$

*23·62. $\vdash : R \dot{\subset} S . \equiv . R \dot{\cup} S = S$

*23·621. $\vdash : R \dot{\subset} S . \equiv . R \dot{\cap} S = R$

*23·63. $\vdash . R \dot{\cup} (R \dot{\cap} S) = R$

*23·631. $\vdash . R \dot{\cap} (R \dot{\cup} S) = R$

*23·632. $\vdash : R = S . \supset . R = R \dot{\cap} S$

*23·633. $\vdash : R \dot{\subset} S . \supset . R \dot{\cup} T = (R \dot{\cap} S) \dot{\cup} T$

*23·64. $\vdash :. R \dot{\subset} T . \vee . S \dot{\subset} T : \supset . R \dot{\cap} S \dot{\subset} T$

*23·65. $\vdash :. R \dot{\subset} S . \vee . R \dot{\subset} T : \supset . R \dot{\subset} S \dot{\cup} T$

*23·66. $\vdash : R \dot{\subset} S . \supset . R \dot{\cup} T \dot{\subset} S \dot{\cup} T$

*23·68. $\vdash . (R \dot{\cap} S) \dot{\cup} (R \dot{\cap} T) = R \dot{\cap} (S \dot{\cup} T)$

*23·69. $\vdash . (R \dot{\cup} S) \dot{\cap} (R \dot{\cup} T) = R \dot{\cup} (S \dot{\cap} T)$

*23·7. $\vdash . (R \dot{\cup} S) \dot{\cup} T = R \dot{\cup} (S \dot{\cup} T)$

*23·71. $R \dot{\cup} S \dot{\cup} T = (R \dot{\cup} S) \dot{\cup} T$ Df

*23·72. $\vdash : P \dot{\subset} R . Q \dot{\subset} S . \supset . P \dot{\cup} Q \dot{\subset} R \dot{\cup} S$

*23·73. $\vdash : P = R . Q = S . \supset . P \dot{\cup} Q = R \dot{\cup} S$

*23·74. $\vdash : P \dot{\cap} Q \dot{\subset} R . P \dot{\cap} R \dot{\subset} Q . \equiv . P \dot{\cap} Q = P \dot{\cap} R$

*23·8. $\vdash . \dot{-} (\dot{-} R) = R$

*23·81. $\vdash : R \dot{\subset} S . \equiv . \dot{-} S \dot{\subset} \dot{-} R$

*23·811. $\vdash : R \dot{\subset} \dot{-} S . \equiv . S \dot{\subset} \dot{-} R$

*23·82. $\vdash : R \dot{\cap} S \dot{\subset} T . \equiv . R \dot{-} T \dot{\subset} \dot{-} S$

*23·83. $\vdash : R = S . \equiv . \dot{-} R = \dot{-} S$

*23·831. $\vdash : R = \dot{-} S . \equiv . S = \dot{-} R$

*23·84. $\vdash . \dot{-} (R \dot{\cap} S) = \dot{-} R \dot{\cup} \dot{-} S$

*23·85. $\vdash . R \dot{\cap} S = \dot{-} (\dot{-} R \dot{\cup} \dot{-} S)$

*23·86. $\vdash . \dot{-} (\dot{-} R \dot{\cap} \dot{-} S) = R \dot{\cup} S$

*23·87. $\vdash . \dot{-} R \dot{\cap} \dot{-} S = \dot{-} (R \dot{\cup} S)$

*23·88. $\vdash . (x, y) . x (R \dot{\cup} \dot{-} R) y$

*23·89. $\vdash . (x, y) . \sim \{ x (R \dot{-} R) y \}$

*23·9. $\vdash . (R \dot{\cup} S) \dot{-} S = R \dot{-} S$

*23·91. $\vdash . R \dot{\cup} S = R \dot{\cup} (S \dot{-} R)$

*23·92. $\vdash : R \dot{\subset} S . \supset . S = R \dot{\cup} (S \dot{-} R)$

*23·93. $\vdash . R \dot{-} S = R \dot{-} (R \dot{\cap} S)$

*23·94. $\vdash : (R) . fR . \equiv . (R) . f(\dot{-} R)$

*23·95. $\vdash : (\exists R) . fR . \equiv . (\exists R) . f(\dot{-} R)$