Disjunctive Normal Form - DNF

<literal> = <variable> | ¬ <variable>

<term> = <literal> (∧ <literal>)*

i.e. <literal> ∧ <literal> ∧ <literal> ......

<formula> = <term> (∨ <term>)*

"OR of ANDs"

e.g.

$$F = (A \land \neg B \land \neg C) \lor (A \land B \land C) \lor (\neg A \land \neg B \land C)$$

DNF-SAT is trivial $O(N)$

N is number of variables

Conjunctive Normal Form - CNF

just switch ∧ and ∨ in previous definition

"AND of ORs"

e.g.

$$F = (A \lor \neg B \lor \neg C) \land (A \lor B \lor C) \land (\neg A \lor \neg B \lor C)$$

CNF-SAT is difficult $O(2^N)$

But De Morgan's theorems allow you to convert each into the other.

So does plain old associativity.

$(A \vee \bar{B} \vee \bar{C}) \wedge (A \vee B \vee C) \wedge (\bar{A} \vee \bar{B} \vee C)$ in CNF

$$(A \vee B \vee C) \wedge (\bar{A} \vee \bar{B} \vee C)$$

$$= A \wedge (\bar{A} \vee \bar{B} \vee C) \vee B \wedge (\bar{A} \vee \bar{B} \vee C) \vee C \wedge (\bar{A} \vee \bar{B} \vee C)$$

$$= \underbrace{A \wedge \bar{A}}_{F} \vee A \wedge \bar{B} \vee A \wedge C \vee B \wedge \bar{A} \vee \underbrace{B \wedge \bar{B}}_{F} \vee B \wedge C \vee C \wedge \bar{A} \vee C \wedge \bar{B} \vee \underbrace{C \wedge C}_{C}$$

$$= A\bar{B} \vee AC \vee B\bar{A} \vee BC \vee C\bar{A} \vee C\bar{B}$$

$\therefore (A \vee \bar{B} \vee \bar{C}) \wedge$ that

$= A \wedge$ that $\vee \bar{B} \wedge$ that $\vee \bar{C} \wedge$ that

$$= AA\bar{B} \vee AAC \vee \underbrace{AB\bar{A}}_{F} \vee ABC \vee \underbrace{AC\bar{A}}_{F} \vee AC\bar{B}$$
$$\vee \bar{B}A\bar{B} \vee \bar{B}AC \vee \underbrace{\bar{B}BA}_{F} \vee \underbrace{\bar{B}BC}_{F} \vee \bar{B}C\bar{A} \vee \bar{B}C\bar{B}$$
$$\vee \bar{C}A\bar{B} \vee \underbrace{\bar{C}AC}_{F} \vee \bar{C}BA \vee \underbrace{\bar{C}BC}_{F} \vee \underbrace{\bar{C}C\bar{A}}_{F} \vee \underbrace{\bar{C}C\bar{B}}_{F}$$

all $= A \wedge \bar{B} \vee A \wedge C \vee A \wedge B \wedge C \vee AC\bar{B} \vee \bar{B}A \vee \bar{B} \wedge A \wedge C \vee$
$\bar{B} \wedge C \wedge A \vee \bar{B}C \vee \bar{C}A\bar{B} \vee \bar{C}BA$ in DNF

If the conversion between domains is non-P, the transformation is not helpful.