The most efficient way to find the product of matrices $M_1 \times M_2 \times M_3 \times M_4 \times \ldots \times M_N$

M_i has $Rows_i$ rows and $Cols_i$ columns

In calculating A×B,

it is required that number of columns in A = number of rows in B, and
the total effort is rows(A)×columns(A)×columns(B) multiplications;
the total number of multiplications is to be minimised.

The individual matrices M_i are given, we do not have to calculate them.

$P_{i,j}$ represents the product of matrices M_i to M_j

e.g. $P_{2,4} = M_2 \times M_3 \times M_4$

$P_{i,j}$ has $Rows_i$ rows and $Cols_j$ columns

$P_{i,i}$ is of course just the matrix M_i

$C_{i,j}$ is the cost (minimum number of multiplications) of calculating $P_{i,j}$.

$C_{i,i}$ is 0

Consider calculating $P_{3,7}$, it could be calculated as

$P_{3,3} \times P_{4,7}$
or $P_{3,4} \times P_{5,7}$
or $P_{3,5} \times P_{6,7}$
or $P_{3,6} \times P_{7,7}$

The cost of calculating:

$P_{3,3} \times P_{4,7}$ is $C_{3,3} + C_{4,7} + Rows_3 \times Cols_3 \times Cols_7$
$P_{3,4} \times P_{5,7}$ is $C_{3,4} + C_{5,7} + Rows_3 \times Cols_4 \times Cols_7$
$P_{3,5} \times P_{6,7}$ is $C_{3,5} + C_{6,7} + Rows_3 \times Cols_5 \times Cols_7$
$P_{3,6} \times P_{7,7}$ is $C_{3,6} + C_{7,7} + Rows_3 \times Cols_6 \times Cols_7$

i.e. C3,7 = min($C_{3,3} + C_{4,7} + Rows_3 \times Cols_3 \times Cols_7$,
$C_{3,4} + C_{5,7} + Rows_3 \times Cols_4 \times Cols_7$,
$C_{3,5} + C_{6,7} + Rows_3 \times Cols_5 \times Cols_7$,
$C_{3,6} + C_{7,7} + Rows_3 \times Cols_6 \times Cols_7$)

The table for N = 9

$C_{1,1}=0$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$	$C_{1,5}$	$C_{1,6}$	$C_{1,7}$	$C_{1,8}$	$C_{1,9}$
	$C_{2,2}=0$	$C_{2,3}$	$C_{2,4}$	$C_{2,5}$	$C_{2,6}$	$C_{2,7}$	$C_{2,8}$	$C_{2,9}$
		$C_{3,3}=0$	$C_{3,4}$	$C_{3,5}$	$C_{3,6}$	$C_{3,7}$	$C_{3,8}$	$C_{3,9}$
			$C_{4,4}=0$	$C_{4,5}$	$C_{4,6}$	$C_{4,7}$	$C_{4,8}$	$C_{4,9}$
				$C_{5,5}=0$	$C_{5,6}$	$C_{5,7}$	$C_{5,8}$	$C_{5,9}$
					$C_{6,6}=0$	$C_{6,7}$	$C_{6,8}$	$C_{6,9}$
						$C_{7,7}=0$	$C_{7,8}$	$C_{7,9}$
							$C_{8,8}=0$	$C_{8,9}$
								$C_{9,9}=0$

$C_{1,9}$ is the minimum cost of calculating $P_{1,9} = P_{1,N} = M_1 \times M_2 \times M_3 \times M_4 \times \ldots \times M_N$