

Difficult Problem, differential equation in y and x

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0;$$

when $x=0$, $\frac{dy}{dx} = 1$
and $y = 3$

$$\mathcal{L}y = Y$$

$$\mathcal{L}\frac{dy}{dx} = sY - y_0$$

$$\mathcal{L}\frac{d^2y}{dx^2} = s^2Y - sy_0 - \frac{dy}{dx}\bigg|_0$$

Laplace transform

$$Y(s^2 + 4s + 3) - (3s + 13) = 0$$

Different problem, algebraic, we can solve it in Y and s

$$Y = \frac{3s + 13}{(s + 3)(s + 1)}$$

$$Y = \frac{5}{s + 1} - \frac{2}{s + 3}$$

$$\mathcal{L}^{-1}\frac{a}{s + b} = ae^{-bx}$$

Inverse Laplace transform

Back to algebraic world in y and x

$$y = 5e^{-x} - 2e^{-3x}$$

Solution!