

By definition, aleph-zero

$$\aleph_0$$

is the number of natural numbers, specifically

$$\aleph_0 = \text{card}(\mathbb{N}) = \#\mathbb{N}$$

It doesn't matter exactly what that means, it is the definition. Aleph-zero is not a number in the usual sense (ordinal), it is a cardinal number and they have different laws of arithmetic.

It is not the same as ∞ , which is no sort of number at all.

So far, we have no reason to suspect that there are any other genuine infinities, and if there are, no reason to suppose that this is the smallest one.

For ordinal numbers, ω_0 is used to represent the first number that you could never reach by counting from 0. It does obey the normal laws of arithmetic, if you know how to apply them.