Average length of a linked list $= \lambda = \frac{n}{h} = \frac{\text{number of items}}{\text{number of lists}}$

Probability that any given list has length $x = P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

For an unsuccessful search, or an insertion

length X	Time to process list $T(X)$	Probability $P(x)$	Product $P(x) \times T(x)$
0	0	$e^{-\lambda}$	0
1	1	$e^{-\lambda}\lambda$	$e^{-\lambda}\lambda$
2	2	$\frac{1}{2}e^{-\lambda}\lambda^2$	$\frac{2}{2}e^{-\lambda}\lambda^2$
3	3	$\frac{1}{6}e^{-\lambda}\lambda^3$	$\frac{3}{6}e^{-\lambda}\lambda^3$
4	4	$\frac{1}{24}e^{-\lambda}\lambda^4$	$\frac{4}{24}e^{-\lambda}\lambda^4$
5	5	$\frac{1}{120}e^{-\lambda}\lambda^4$	$\frac{5}{120}e^{-\lambda}\lambda^5$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

unsuccessful search, or insertion

$$\text{Expected time} = \frac{\sum(P(x) \cdot T(x))}{\sum(P(x))}$$

$\sum(P(x)) = 1$: sum of all probabilities, by definition.

$$\sum(P(x) \cdot T(x)) = \frac{1}{1!}e^{-\lambda}\lambda^1 + \frac{2}{2!}e^{-\lambda}\lambda^2 + \frac{3}{3!}e^{-\lambda}\lambda^3 + \frac{4}{4!}e^{-\lambda}\lambda^4 + \ldots\ldots$$

$$= \frac{1}{0!}e^{-\lambda}\lambda^1 + \frac{1}{1!}e^{-\lambda}\lambda^2 + \frac{1}{2!}e^{-\lambda}\lambda^3 + \frac{1}{3!}e^{-\lambda}\lambda^4 + \ldots\ldots$$

$$= e^{-\lambda}\lambda\left(\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\right)$$

$$= e^{-\lambda}\lambda e^{\lambda}$$

$$= \lambda$$

$$\text{Expected time} = \lambda = \frac{n}{h}$$

Average length of a linked list $= \lambda = \frac{n}{h} = \frac{\text{number of items}}{\text{number of lists}}$

Probability that any given list has length $x = P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

For a successful search

length x	Time to process list $T(x)$	Probability $P(x)$	Product $P(x) \times T(x)$
0	0	0	0
1	$\frac{1}{2}$	$e^{-\lambda}\lambda$	$\frac{1}{2} e^{-\lambda}\lambda$
2	$\frac{2}{2}$	$\frac{1}{2} e^{-\lambda}\lambda^{2}$	$\frac{1}{2}\frac{2}{2} e^{-\lambda}\lambda^{2}$
3	$\frac{3}{2}$	$\frac{1}{6} e^{-\lambda}\lambda^{3}$	$\frac{1}{2}\frac{3}{6} e^{-\lambda}\lambda^{3}$
4	$\frac{4}{2}$	$\frac{1}{24} e^{-\lambda}\lambda^{4}$	$\frac{1}{2}\frac{4}{24} e^{-\lambda}\lambda^{4}$
5	$\frac{5}{2}$	$\frac{1}{120} e^{-\lambda}\lambda^{4}$	$\frac{1}{2}\frac{5}{120} e^{-\lambda}\lambda^{5}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Successful Search

$$\text{Expected time} = \frac{\sum(P(x)T(x))}{\sum(P(x))}$$

This time, the $e^{-\lambda}$ element $P(0)$ is missing,

so $\sum(P(x)) = 1 - e^{-\lambda}$

and $\sum(P(x)T(x))$ is half of what it was before

so expected time $= \dfrac{\frac{1}{2}\lambda}{1-e^{-\lambda}}$

When λ is at all large, >3, e^{λ} is very large
$e^{-\lambda}$ is nearly zero

$$\boxed{\text{large } \lambda \Rightarrow \text{time} = \tfrac{1}{2}\lambda}$$

When λ is small, $e^{-\lambda} = 1 - \lambda + \underbrace{\frac{\lambda^2}{2} - \frac{\lambda^3}{6} + \frac{\lambda^4}{24} \ldots}_{\text{very small}}$

$\approx 1 - \lambda$

so time $= \dfrac{\frac{1}{2}\lambda}{1-(1-\lambda)} = \dfrac{1}{2}$

$$\boxed{\text{small } \lambda \Rightarrow \text{time} = \tfrac{1}{2}}$$