After inserting one new item into a tree that was balanced, we have found a node that is not balanced any more. We are looking at the deepest (closest to the leaves) such node.

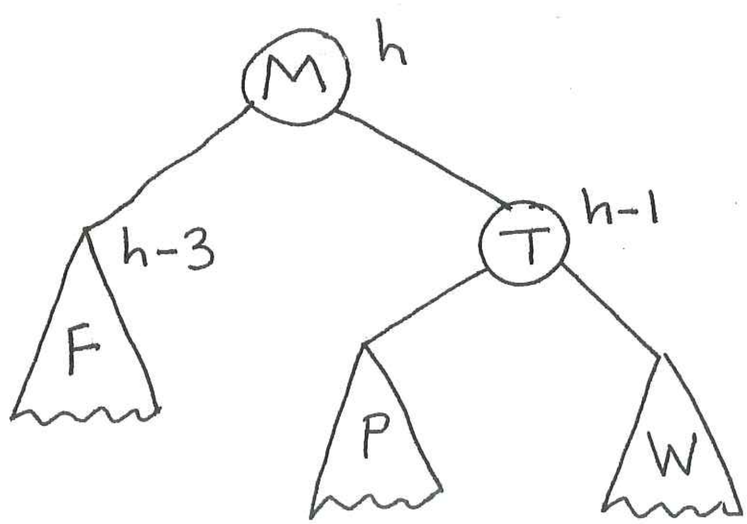
Call it M, and whatever its height is, call that h.

Adding one item can change height be at most + or - 1.
Therefore can change balance by at most + or - 1.
M was balanced before, and now is not, therefore balance now must be either -2 or +2.

1a. Consider the case of +2 first

M's balance is +2, so right subtree is deepest.
M's height is h, so right subtree's height is h-1,
and left subtree's height must be h-3.

Left subtree can not be less than empty, so h-3 must be at least 0,
so h-1, height of right subtree, must be at least 2.
We can rely on right subtree having a top node, call it T,
and if we care, T must have at least one node below it, to the left, to the right, or both.

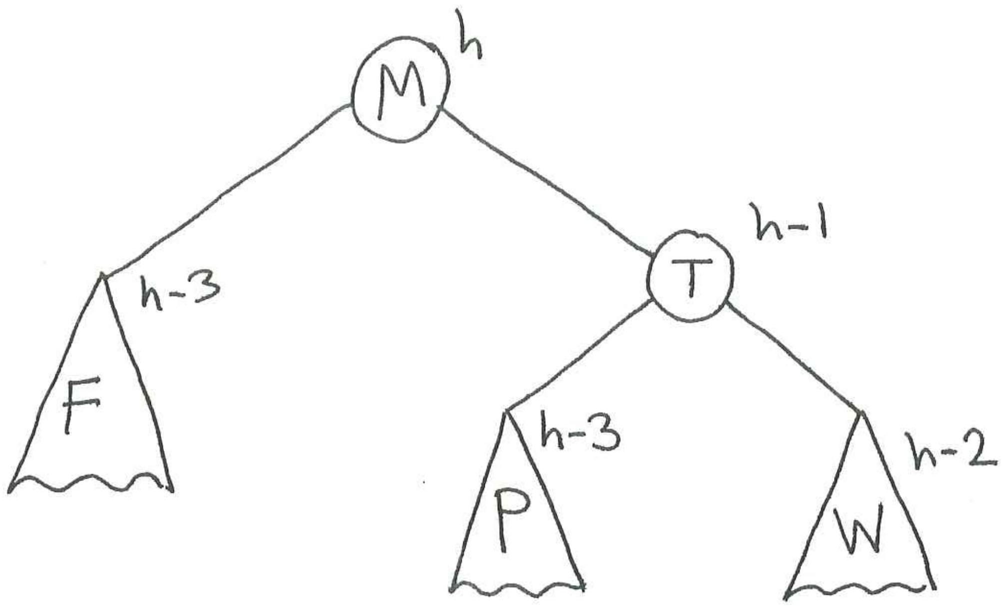


Because T's height is h-1 and T is balanced, one of its subtrees P or W must have a height of h-2, and the other must have a height of either h-2 or h-3.

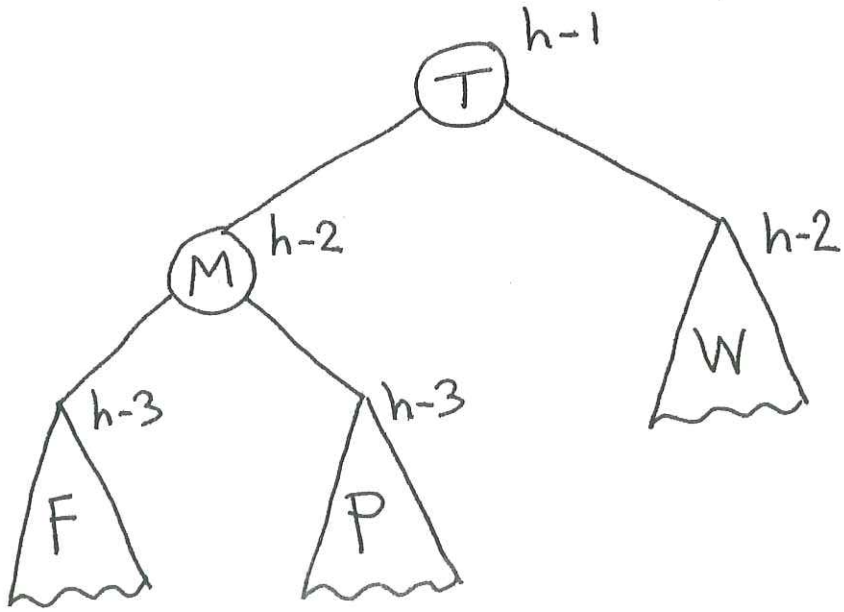
Only one item has been inserted, so only one of the subtrees can possibly have grown. If they have both now got a height of h-2, then one of them must already have had height h-2 before the insertion. That would mean that T's height must already have been h-1, which would mean that M was already unbalanced. All nodes were balanced before the insertion, so this can not be the case.

Either P has height h-3 and W has height h-2
or P has height h-2 and W has height h-3.

2a. Consider the first case first



In this case, a simple rearrangement of the links bringing T up above M, leaves all the content in the same order

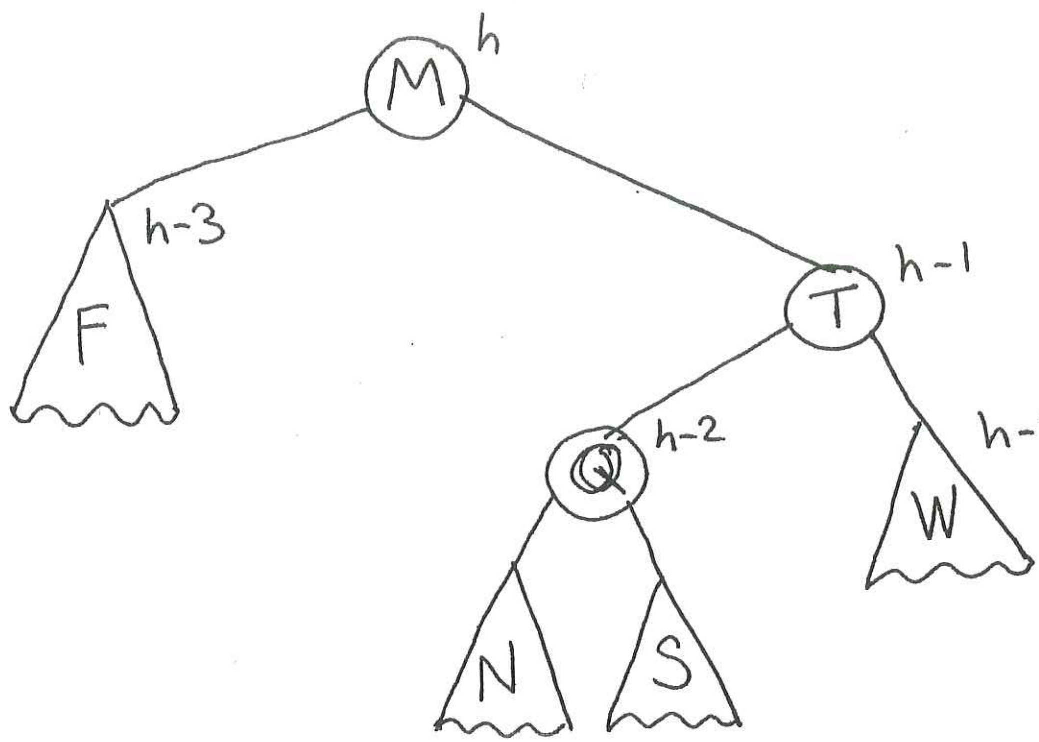


Here, M has two subtrees of height h-3, so it is balanced and has height h-2, so now T has two subtrees of height h-2 and is balanced, and the whole tree has been reduced to height h-1.

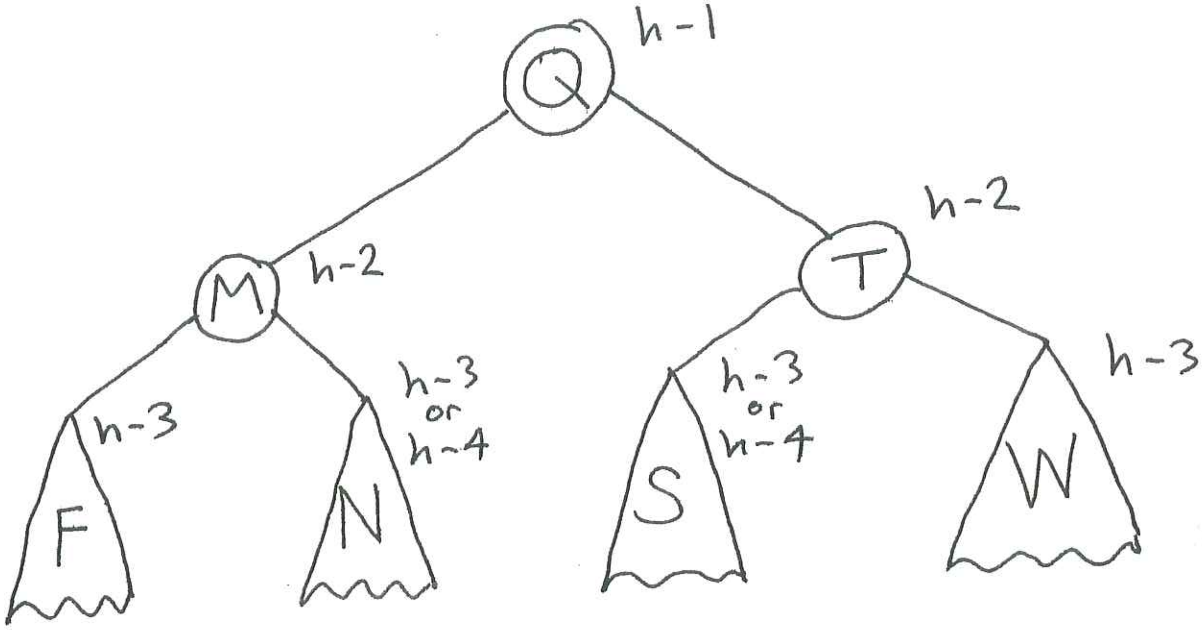
2b. Now the other case
P has height h-2 and W has height h-3

Note that the same rearrangement would not succeed in rebalancing the tree.

But we already know that the larger of T's subtrees (in this case P) must have at least one node in it, so we can redraw the picture in more detail, calling that node Q and its subtrees N and S:



At least one out of N and S must have height h-3 (or Q's height would not be h-2 which we know it is), and the other has height h-3 or h-4 because we know Q is balanced. In either case, a rearrangement in which Q is brought to the top:



Here, M has a subtree of height h-3 and another of h-3 or h-4. Either way it is balanced, and has a height of h-2. T is in the same situation, so Q has two subtrees of height h-2 and is therefore balanced, and the overall height is back down to h-1.

1b. The remaining half.

All of these deductions were based on M's balance being +2. For the case of -2, there is no extra work needed. Just imagine all the diagrams reflected in a mirror. All positive balances become negative and vice versa, but everything else is the same.