Assume that an enumeration of real numbers $0 \le x < 1$ exists.

Let E be such an enumeration, $E = \langle E_0, E_1, E_2, E_3, E_4, \ldots \rangle$ where E_i is the i^{th} number in the enumeration.

Now think of each of those numbers in Binary, we assume a "0." in front of each to make the range be $0 \le x < 1$.

We couldn't write out the table because it is infinite in both directions: there are an infinite number of E_is in the enumeration, and being real numbers, many of them are infinite in length.

In fact, let's insist that they are all infinite in length. If one is naturally finite, like 3/8: 0.011, we'll just fill it out with an endless stream of extra zeros at the end: 0.01100000000000000000000...

If N is a number, let $N \mid j$ represent the j^{th} digit of N, with $N \mid 0$ being the digit immediately following the "decimal point".

So without making any assumptions about what the enumeration is, we can write it out as

$E_0 =$	0	.	$E_0 \mid 0$	$E_0 \mid 1$	$E_0 \mid 2$	$E_0 \mid 3$	$E_0 \mid 4$	$E_0 \mid 5$	$E_0 \mid 6$	$E_0 \mid 7$	$E_0 \mid 8$	$E_0 \mid 9$	...
$E_1 =$	0	.	$E_1 \mid 0$	$E_1 \mid 1$	$E_1 \mid 2$	$E_1 \mid 3$	$E_1 \mid 4$	$E_1 \mid 5$	$E_1 \mid 6$	$E_1 \mid 7$	$E_1 \mid 8$	$E_1 \mid 9$	...
$E_2 =$	0	.	$E_2 \mid 0$	$E_2 \mid 1$	$E_2 \mid 2$	$E_2 \mid 3$	$E_2 \mid 4$	$E_2 \mid 5$	$E_2 \mid 6$	$E_2 \mid 7$	$E_2 \mid 8$	$E_2 \mid 9$	...
$E_3 =$	0	.	$E_3 \mid 0$	$E_3 \mid 1$	$E_3 \mid 2$	$E_3 \mid 3$	$E_3 \mid 4$	$E_3 \mid 5$	$E_3 \mid 6$	$E_3 \mid 7$	$E_3 \mid 8$	$E_3 \mid 9$	...
$E_4 =$	0	.	$E_4 \mid 0$	$E_4 \mid 1$	$E_4 \mid 2$	$E_4 \mid 3$	$E_4 \mid 4$	$E_4 \mid 5$	$E_4 \mid 6$	$E_4 \mid 7$	$E_4 \mid 8$	$E_4 \mid 9$	...
$E_5 =$	0	.	$E_5 \mid 0$	$E_5 \mid 1$	$E_5 \mid 2$	$E_5 \mid 3$	$E_5 \mid 4$	$E_5 \mid 5$	$E_5 \mid 6$	$E_5 \mid 7$	$E_5 \mid 8$	$E_5 \mid 9$	...
...	...	...	...	...	...	...	...	...	...	...	...		

Now I can construct a number X, where $X \mid i = \text{not}(E_i \mid i)$

$$X = 0 \;.\; \text{not}(E_0 \mid 0)\ \ \text{not}(E_1 \mid 1)\ \ \text{not}(E_2 \mid 2)\ \ \text{not}(E_3 \mid 3)\ \ \text{not}(E_4 \mid 4)\ \ \text{not}(E_5 \mid 5)\ \ldots$$

If X is in the enumeration, then X must = E_j for some j, and if two numbers are the same, all their digits are the same.

But $X \ne E_0$ because $X \mid 0 \ne E_0 \mid 0$
and $X \ne E_1$ because $X \mid 1 \ne E_1 \mid 1$
and $X \ne E_2$ because $X \mid 2 \ne E_2 \mid 2$
and $X \ne E_3$ because $X \mid 3 \ne E_3 \mid 3$ ... etc.

In fact X is different from all of the E_i in at least one digit position.
Therefore X is not in the enumeration.
But X is a real number, $0 \le x < 1$.
Therefore E is not an enumeration of real numbers $0 \le x < 1$.
That is deduced from a single assumption, that E *is* such an enumeration.