

$S = \{a_1, \dots, a_n\}$  jobs to be scheduled

$s_i =$  start time of job  $i$

$f_i =$  finish

sorted so that  $i \leq j \Leftrightarrow f_i \leq f_j$

$S_{ij} =$  all jobs that start after  $a_i$  is done  
end before  $a_j$  starts

$$= \{a_k \in S \mid f_i \leq s_k < f_k \leq s_j\}$$

add fake jobs  $a_0, a_{n+1}$

$$f_0 = 0$$

$$s_{n+1} = \infty$$

$$S_{ij} = \{\} \text{ if } i \geq j$$

$S_{0, n+1}$  represents the whole problem

$S_{ij}$  (where  $i < j$ ) represents a sub-problem

if the solution to  $S_{ij}$  contains  $a_k$

then we will have to solve sub-problems  $S_{ik}$  and  $S_{kj}$

solution to  $S_{ij}$  is then  $(\text{solution to } S_{ik}) \cup (\text{sol to } S_{kj}) \cup \{a_k\}$

$A[i][j]$  is to hold the number of jobs in the biggest subset  
of compatible jobs in  $S_{ij}$

if  $a_k$  appears in the solution to  $S_{ij}$

$$\text{then } A[i][j] = A[i][k] + 1 + A[k][j]$$

try all possible values of  $k$

$$A[i][j] = \begin{cases} 0 & \text{if } S_{ij} = \{\} \\ \max_{i < k < j} (A[i][k] + 1 + A[k][j]) & \text{otherwise} \end{cases}$$

If  $S_{ij}$  is any sub-problem  
 and  $a_m$  is the job in  $S_{ij}$  with earliest finish time  
 $f_m \leq f_x$  for all  $x$  in  $i \dots j$

then  $S_{im}$  is empty

and  $a_m$  is in the optimal solution to  $S_{ij}$

$\therefore$  to solve  $S_{ij}$ :

take  $a_i$

and then solve  $S_{i+1,j}$

$i:$	0	1	2	3	4	5	6	7	8	9	10	11	12
$S_i:$	-	1	3	0	5	3	5	6	8	8	2	12	$\infty$
$f_i:$	0	4	5	6	7	8	9	10	11	12	13	14	-

Use  $a_1$ , left with  $\{4, 6, 7, 8, 9, 11\}$

Use  $a_4$ , left with  $\{8, 9, 11\}$

Use  $a_8$ , left with  $\{11\}$

Use  $a_{11}$ , left with  $\{\}$ , all done.