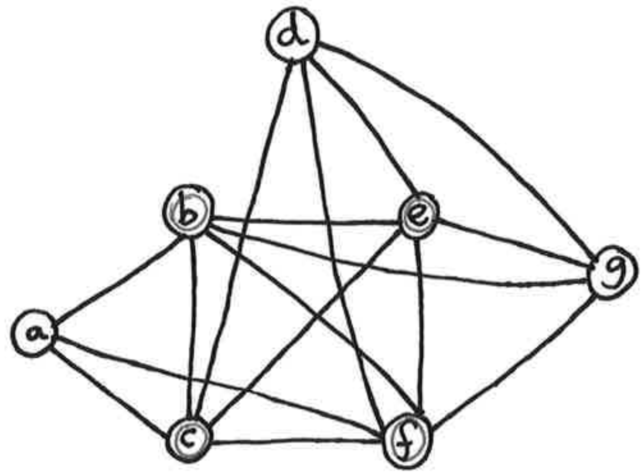
The size of a graph is the number of nodes in it, written $|G|$.

nodes are sometimes called vertices.

edges are sometimes called arcs.

Here is a graph G:



V or N = The nodes of G are $\{a, b, c, d, e, f, g\}$

E = The edges of G are $\{(a,b), (a,c), (a,f), (b,c), \{b,e\}, \{b,f\}, \{b,g\},$
$(c,d), (c,e), (c,f), (d,g), (e,f), (e,g), (f,g)\}$

In this example the edges are all bidirectional, so technically E also contains $(b,a), (c,a), (f,a), \ldots (g,f)$.

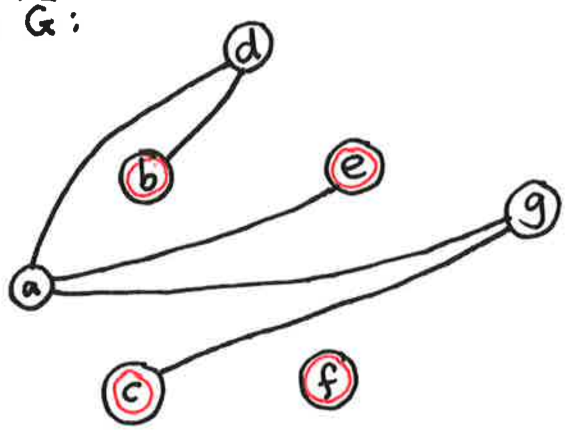
A Clique is a subgraph in which every node is directly connected to every other

$$\forall x, y \in N, \ (x,y) \in E$$

In G, the nodes $\{b, c, e, f\}$ form a clique of size 4.

The complement of a graph, $\overline{G}$, contains all the nodes in G, and all the edges that are <u>not</u> in G.

This is $\overline{G}$:



A Vertex Cover is a set of nodes such that every edge in the graph is connected to at least one node in the cover.

$\{a, d, g\}$ is a vertex cover for $\overline{G}$.

<u>Claim</u>

A graph G of size N has a clique of size K (call it V')

$\Leftrightarrow$

$V - V'$ is a vertex cover (of size $N-K$) of $\overline{G}$.

Demonstration ($\Rightarrow$)

let (x,y) be any edge in $\overline{G}$,

(x,y) can not be in G,

$\therefore$ at least one of x or y is not in the clique

(if they both were, then (x,y) would have to be in G)

$\therefore$ at least one of x or y is in $V-V'$

$\therefore$ $V-V'$ is a vertex cover of $\overline{G}$

The other way ($\Leftarrow$)

Let (x,y) be any edge in $\overline{G}$, and

V^c is a vertex cover of $\overline{G}$, and size $V^c = N-K$

$\therefore$ x is in V^c or y is in V^c or both

$\therefore$ for all x and y in V,

if $x \notin V^c$ and $y \notin V^c$

then $(x,y) \in E$

$\therefore$ $V-V^c$ is a clique