

The clique problem solves 3CNF

Take any formula in 3CNF, it can have any number of terms, N.

Each term has exactly three items.

Each item is either a variable or the negation of a variable.

No variable may appear more than once in any term.

example with N = 4

$$(a \vee b \vee \neg c) \wedge (\neg a \vee \neg c \vee d) \wedge (b \vee c \vee \neg d) \wedge (\neg b \vee c \vee \neg d)$$

The solution is an assignment of a value to every variable that makes the formula true. It is written as a conjunction that involves every variable exactly once. setting a = true, b = true, c = false, d = false makes the example formula true, so the solution is

$$(a \wedge b \wedge \neg c \wedge \neg d)$$

To make the whole formula true, every term must be true.

To make a term true, only one of its items needs to be true (but more than one is still OK)

Rule A:

The solution must include at least one item from each term

Non-rule B:

There is nothing to be gained by having more than one item from any term

Rule C:

The solution must not contain any variable and its negation

We construct a graph  $3 \times N$  nodes, each corresponding with an item in a term. The variable b is the same thing in the first and third terms, but is still represented by a different node each time it appears. The nodes will have the same names as their items, but with a superscript to indicate their term.

The graph is more readable if the nodes for each term are kept close together, but the nodes for different terms are far apart.

$a^1$     $b^1$     $\neg c^1$

$\neg b^4$

$c^4$

$\neg d^4$

$\neg a^2$

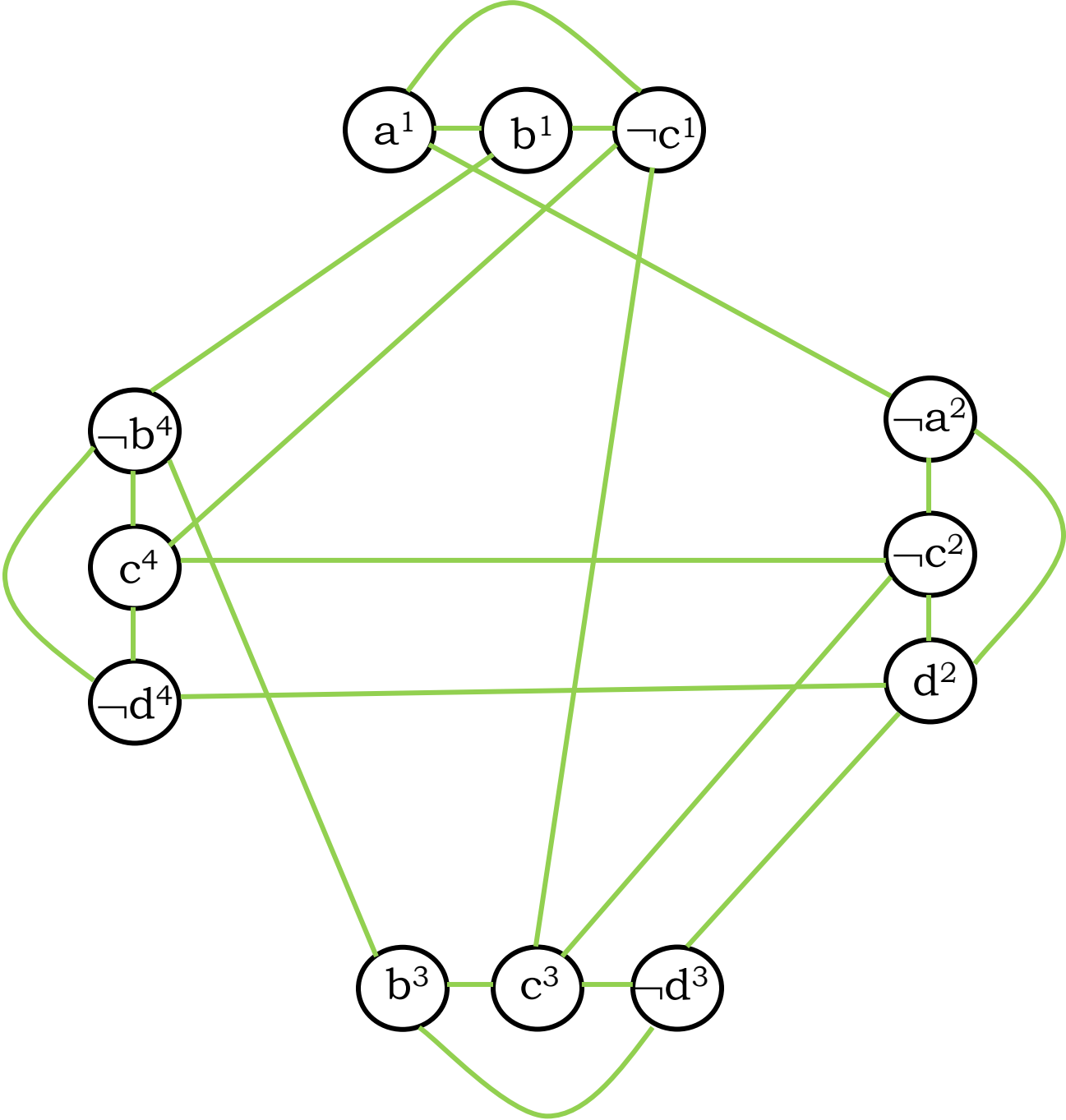
$\neg c^2$

$d^2$

$b^3$     $c^3$     $\neg d^3$

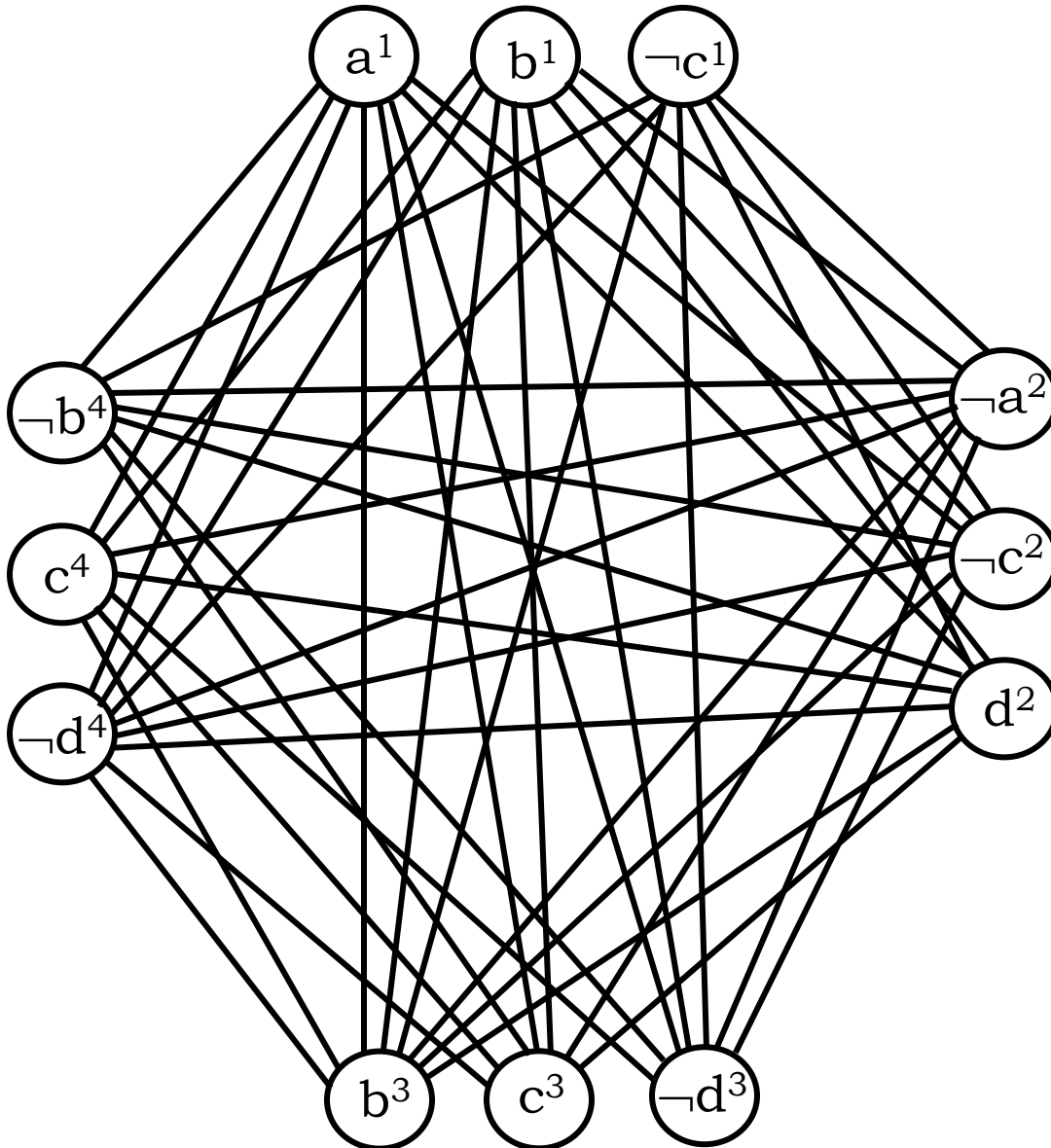
The edges represent rule C and non-rule B. Two nodes are connected if they may both appear in a solution. That is, if they are not each other's negations and they are from different terms.

It is much easier to draw the complement of the graph, just the edges that don't exist:

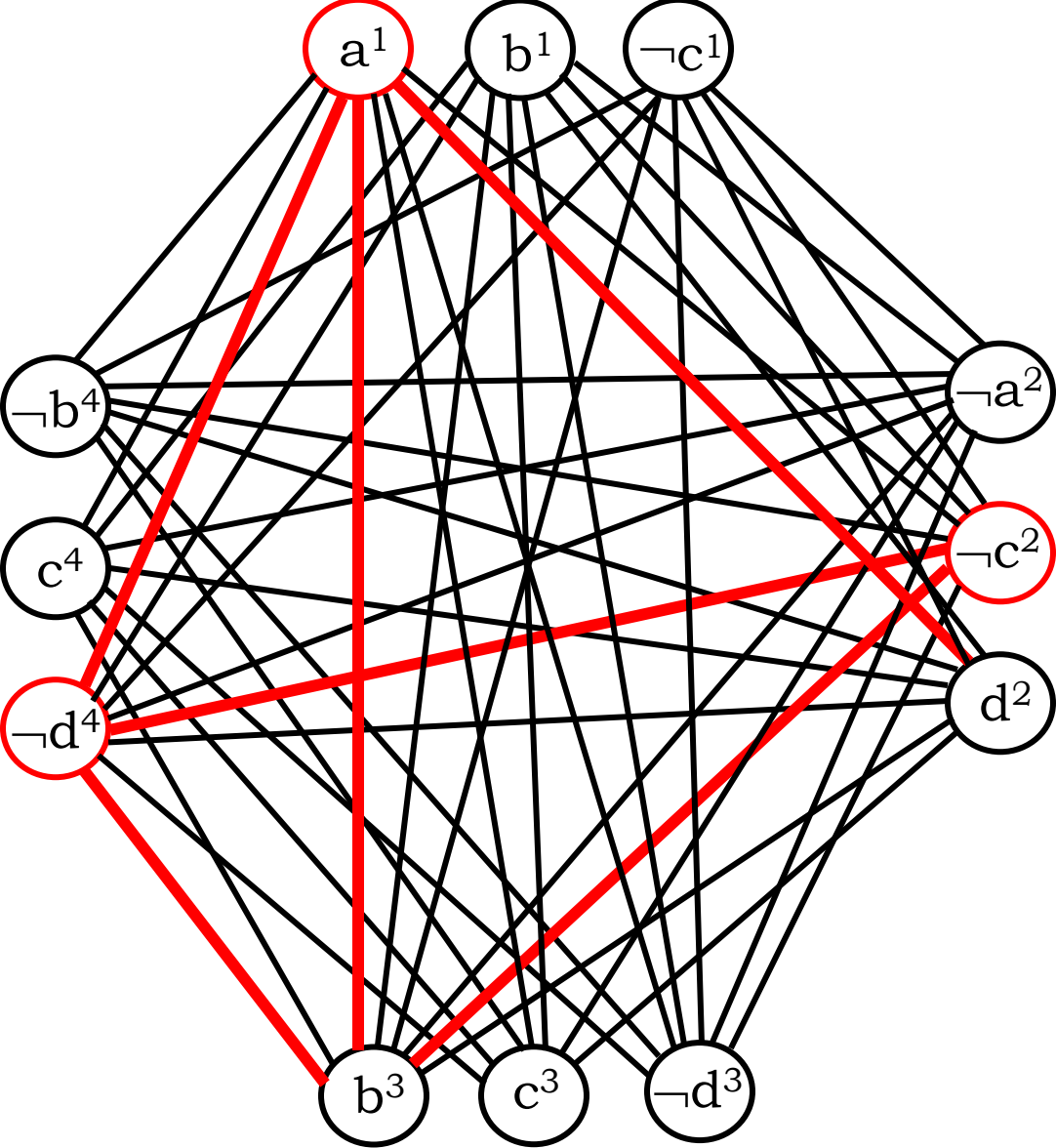


Sadly, the complement of the graph doesn't show what is needed, so ...

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Rule C means that any two connected nodes can appear together in a solution. Rule A tells us that we must find four such nodes, all connected together, in other words a 4-clique:

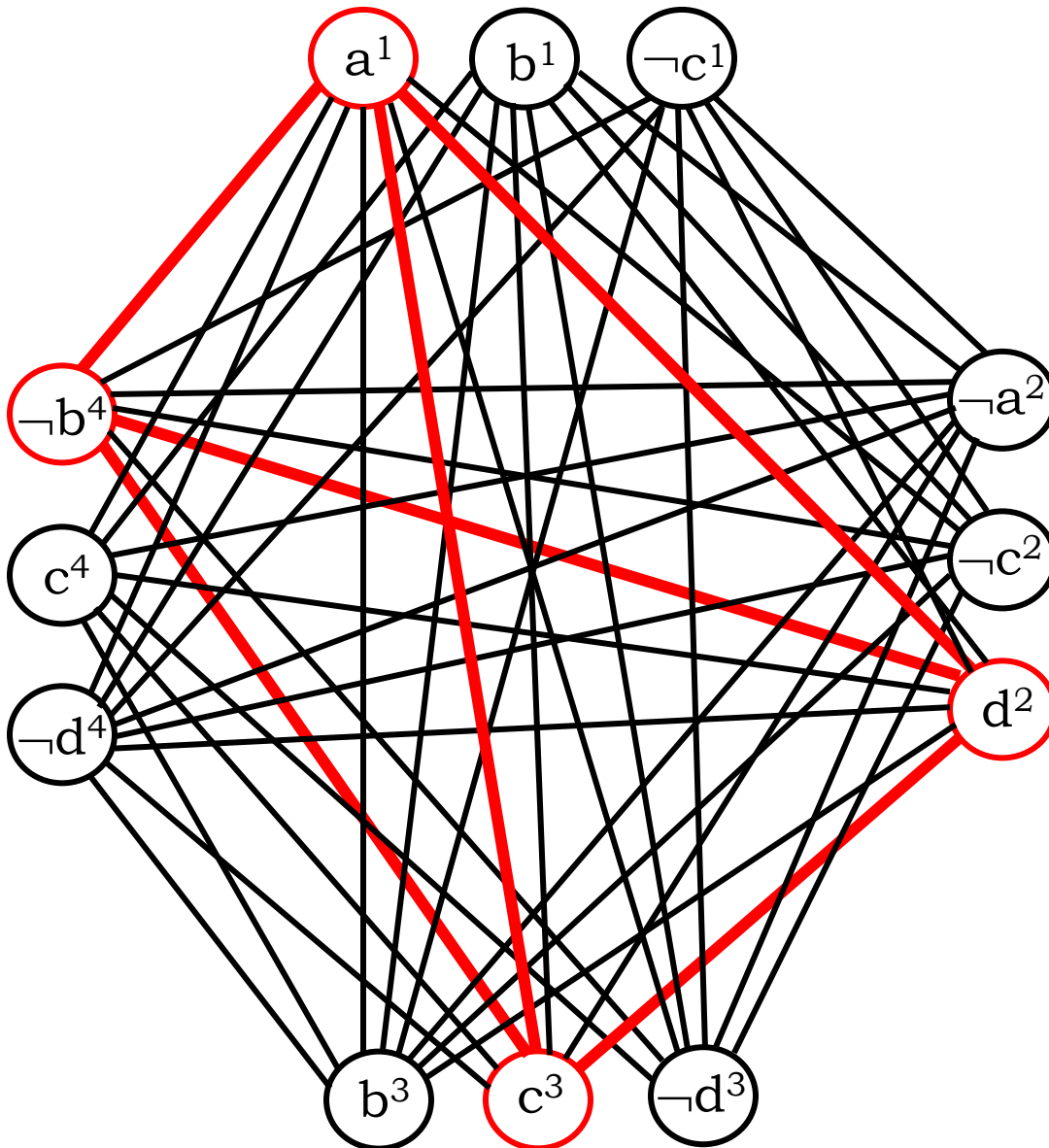


The red 4-clique shows the solution originally given,  $(a \wedge b \wedge \neg c \wedge \neg d)$

But there are many more.

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This one is another easily verified solution,  $(a \wedge \neg b \wedge c \wedge d)$ .

So Clique solves 3CNF.