An Extra Bonus

5 points

to the first person

not to mention

or allude to

any tests

in any way.

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

1. Main is running, remaining input is “ `* + 1 x 3 ?` “

main

where did I come from? nowhere

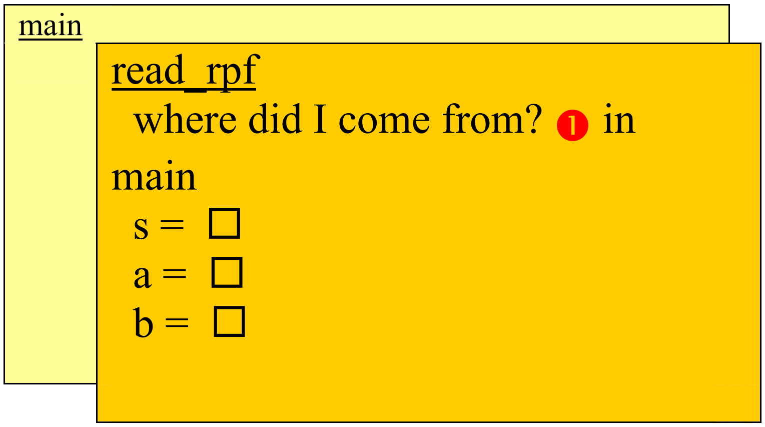
f = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

2. main calls read_rpf at point ❶, remaining input is “ `* + 1 x 3 ?` “



main

read_rpf

where did I come from? ❶ in main

s = □

a = □

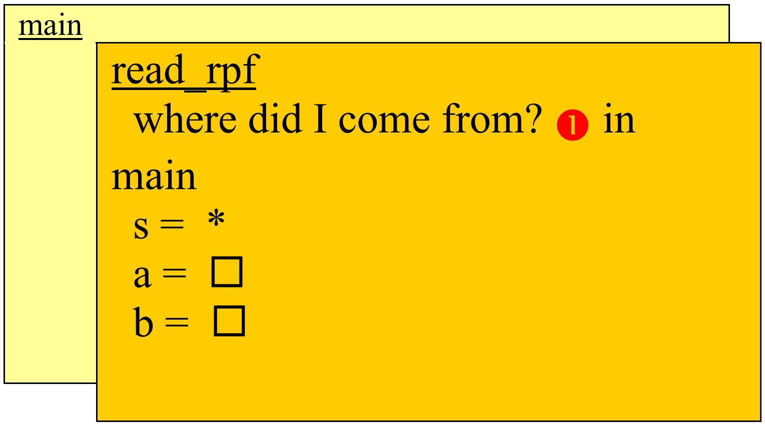
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

3. The symbol is read and we get to point ❷, remaining input is " + 1 x 3 ? "



main

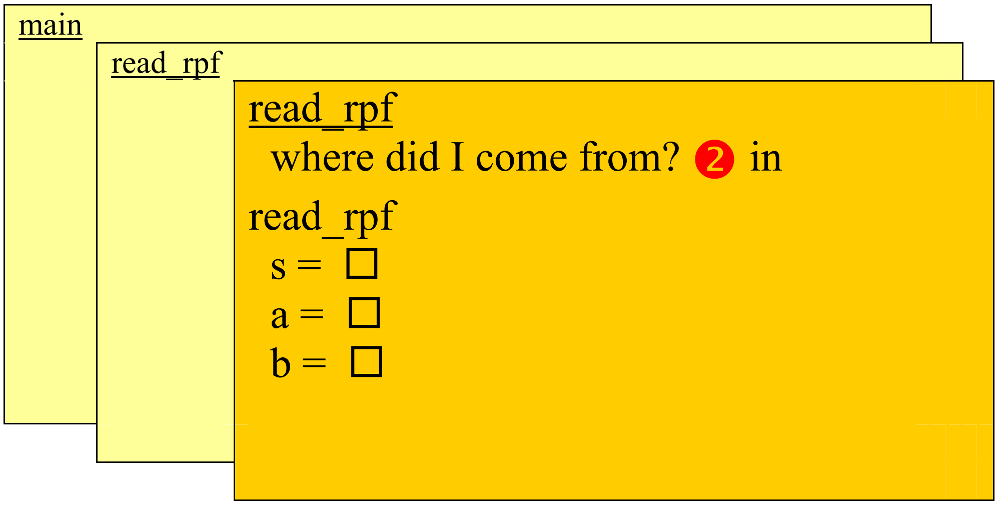
read_rpf
where did I come from? ❶ in main
s = *
a = □
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula(‘N’, s);
  else if (is_variable(s))
    return new Formula(‘X’);
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula(‘C’, s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

4. so read_rpf is called recursively. Remaining input is still “ + 1 x 3 ? “



main

read_rpf

read_rpf

where did I come from? ❷ in

read_rpf

s = □

a = □

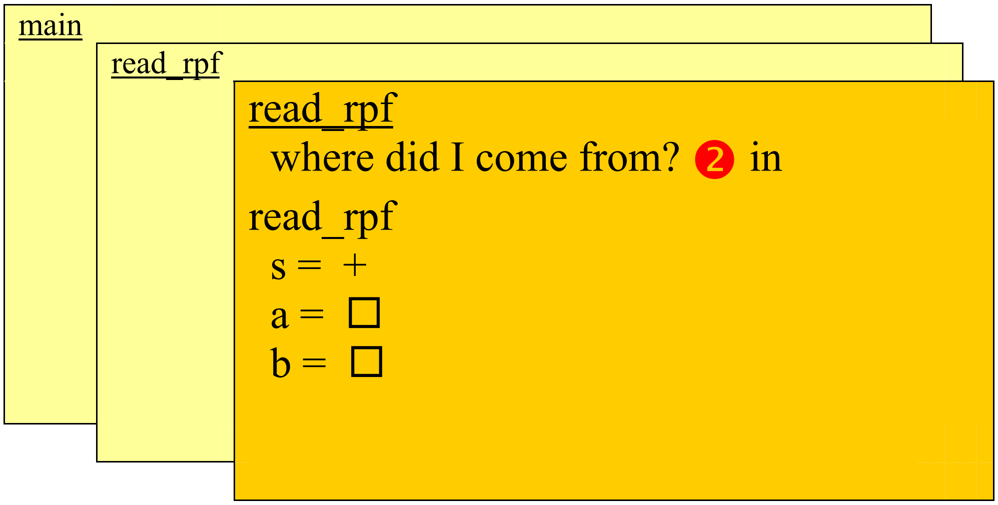
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

5. The symbol is read and we get to point ❷, remaining input is “ `1 x 3 ?` “



main

read_rpf

read_rpf

where did I come from? ❷ in

read_rpf

s = +

a = □

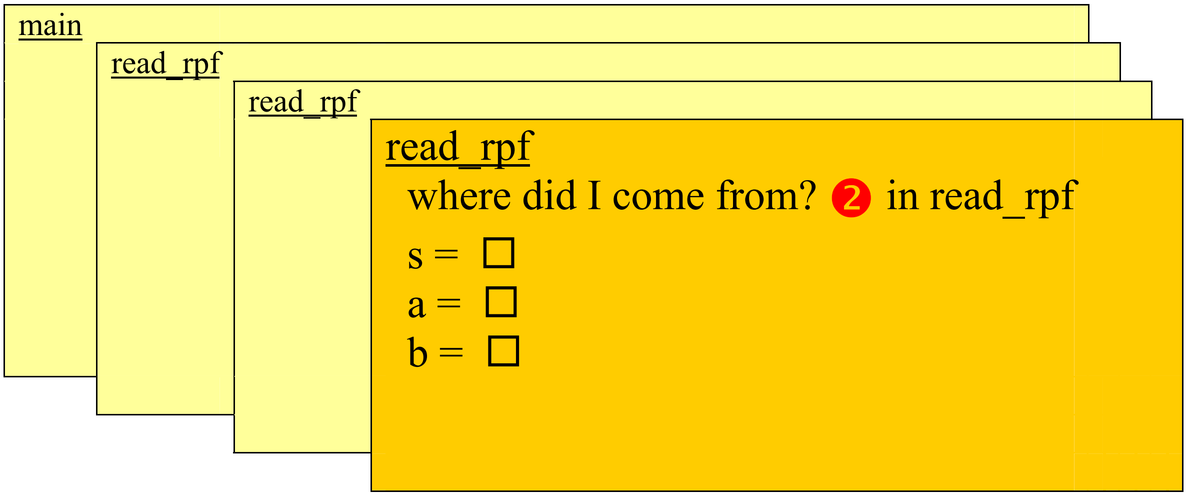
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

6. so read_rpf is called recursively again. Remaining input is still “ 1 x 3 ? “



main

read_rpf

read_rpf

read_rpf

where did I come from? ❷ in read_rpf

s = □

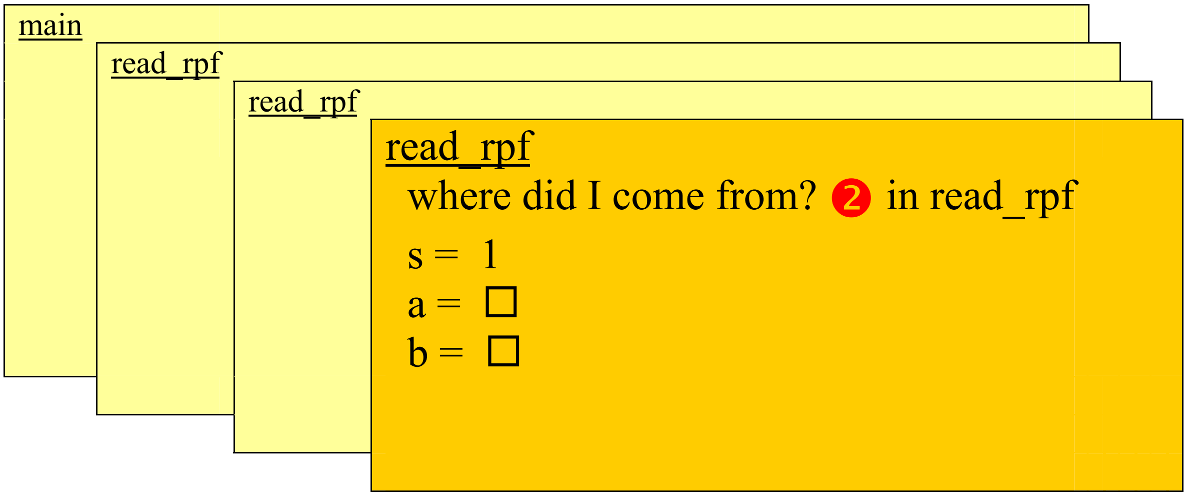
a = □

b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

7. a symbol is read, and seen to be a number. Remaining input: “ x 3 ? “



main

read_rpf

read_rpf

read_rpf

where did I come from? ❷ in read_rpf

s = 1

a = □

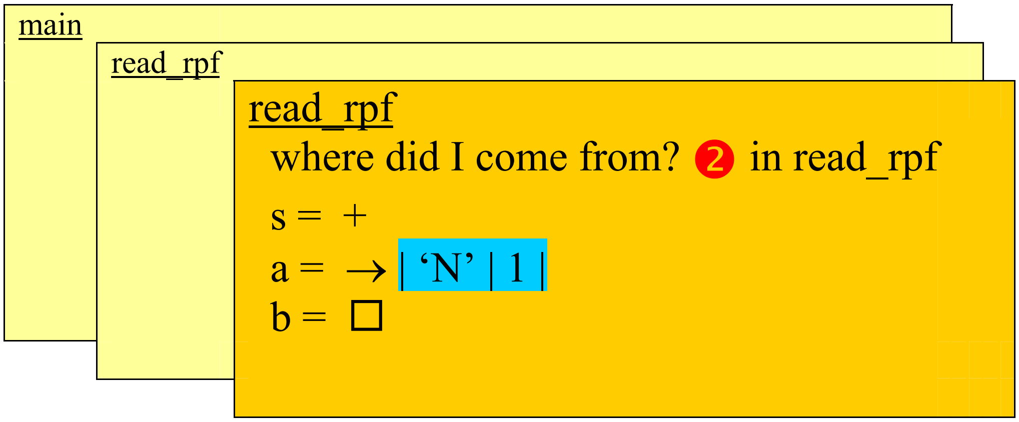
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

8. read_rpf creates a formula and returns it. Remaining input: “ x 3 ? “



main

read_rpf

read_rpf

where did I come from? ❷ in read_rpf

s = +

a = → | ‘N’ | 1 |

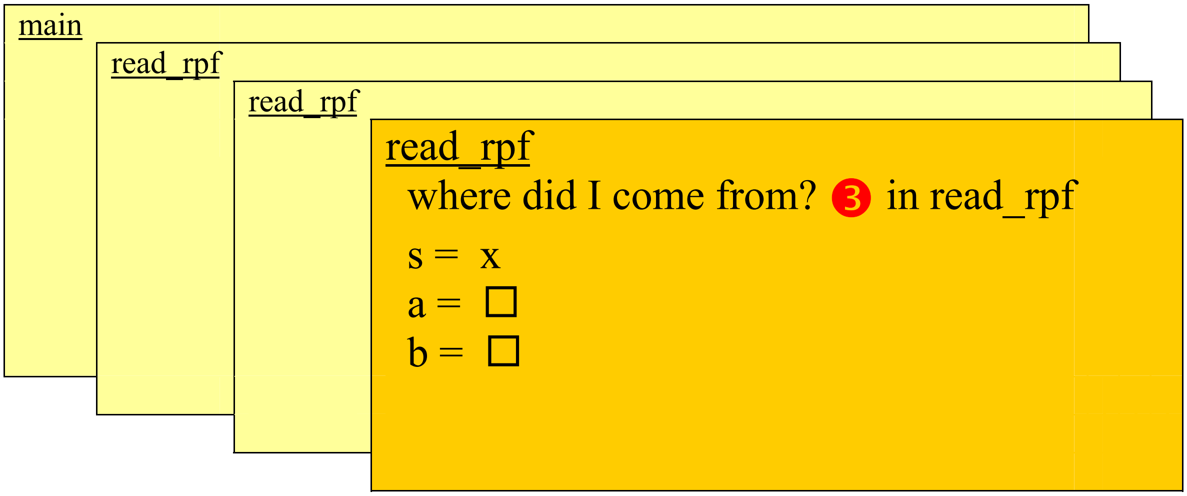
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ② read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ③ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

9. and continues to point ③ for another recursive call.

the next symbol is read, and seen to be a variable.   Remaining input: “ 3 ? “



where did I come from? ③ in read_rpf

s = x

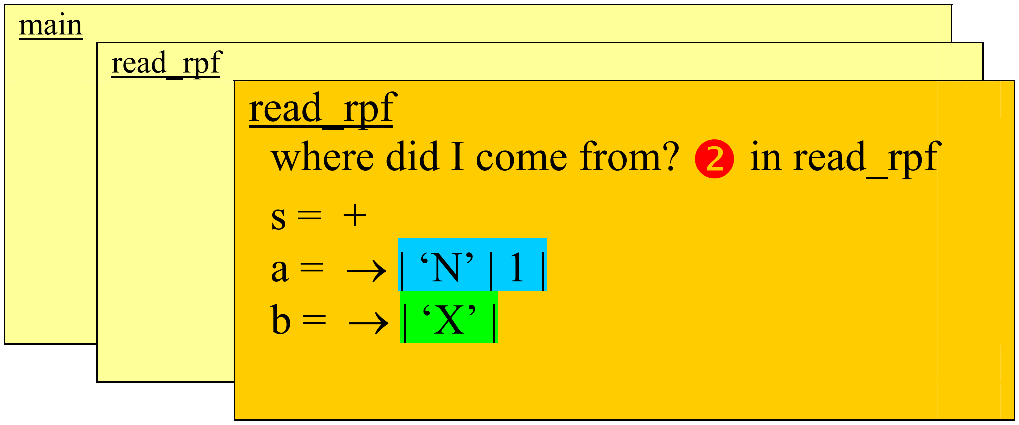
a = □

b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

10. so read_rpf creates a formula and returns it. Remaining input: “ 3 ? “



main

read_rpf

read_rpf

where did I come from? ❷ in read_rpf

s = +

a = → | ‘N’ | 1 |

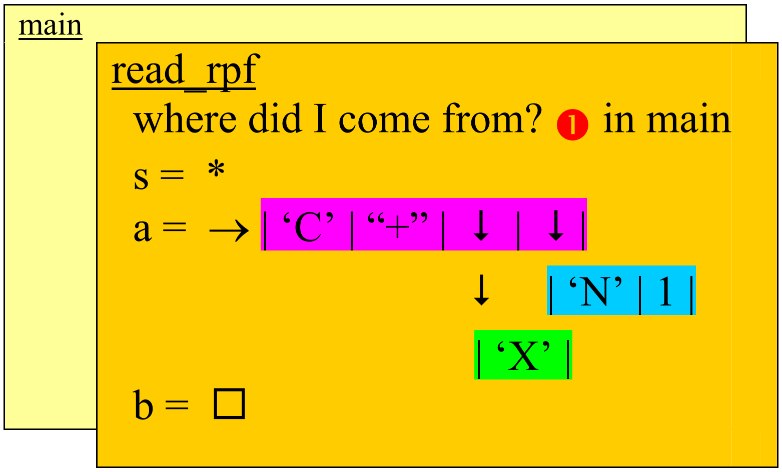
b = → | ‘X’ |

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula(‘N’, s);
  else if (is_variable(s))
    return new Formula(‘X’);
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula(‘C’, s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

11. But note we are back to point 3 in read_rpf.
All it does is create a new complex formula and return it.
This takes us back to point 2, and the remaining input is “ 3 ? “



main

read_rpf

where did I come from? ❶ in main

s = *

a = → | ‘C’ | “+” | ↓ | ↓ |

↓ | ‘N’ | 1 |

| ‘X’ |

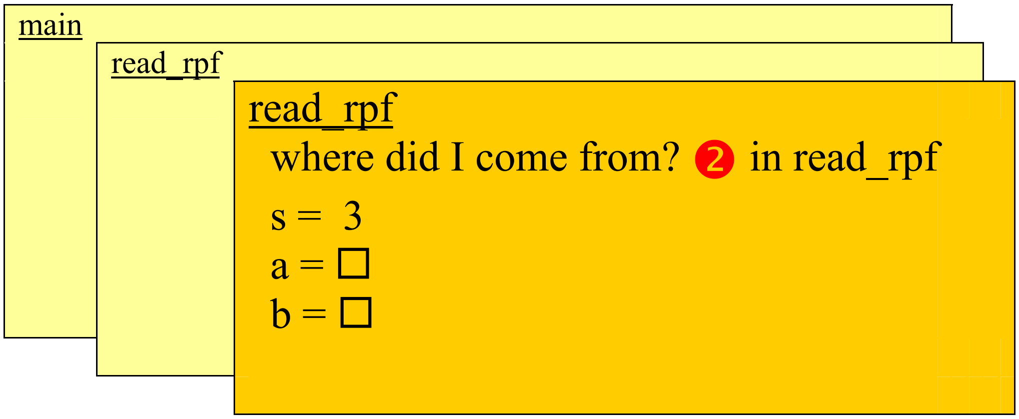
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula(‘N’, s);
  else if (is_variable(s))
    return new Formula(‘X’);
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula(‘C’, s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

12. so we progress again to point ❸ for another recursive call.
    This time, the recursive call reads the “3” and makes it into a formula:



main

read_rpf

read_rpf

where did I come from? ❷ in read_rpf

s = 3

a = □

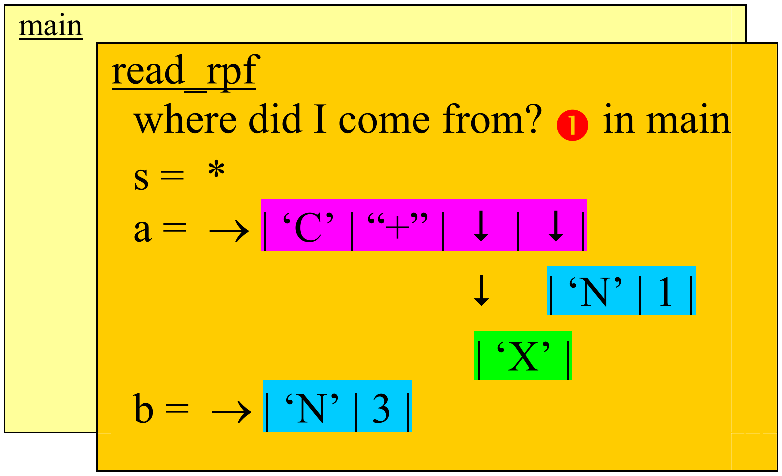
b = □

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula('N', s);
  else if (is_variable(s))
    return new Formula('X');
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula('C', s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

13. we return back to point ❸ with that formula ready to become the value of b.
   Remaining input " ? ".



main

read_rpf

where did I come from? ❶ in main

s = *

a = → | 'C' | "+" | ↓ | ↓ |

↓ | 'N' | 1 |

| 'X' |

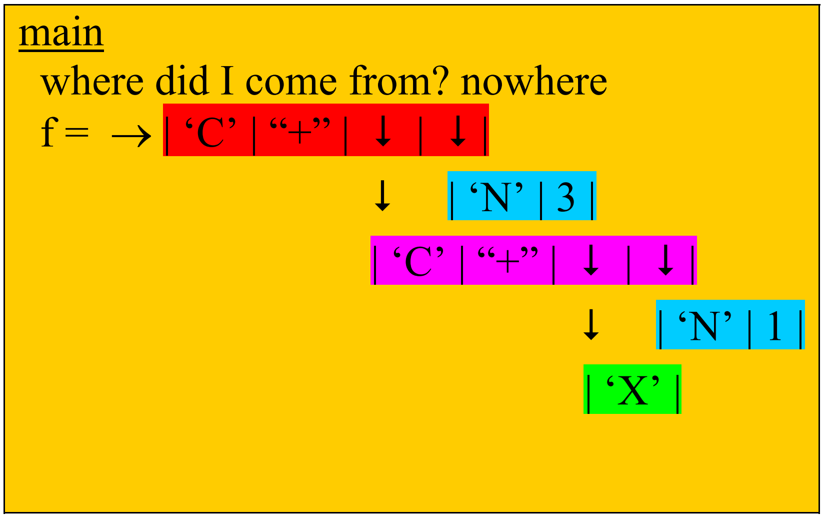
b = → | 'N' | 3 |

```
Formula * read_rpf()
{ string s = read_next_symbol();
  if (is_number(s))
    return new Formula(‘N’, s);
  else if (is_variable(s))
    return new Formula(‘X’);
  else if (is_operator(s))
  { Formula * a = ❷ read_rpf();
    if (a==NULL) return NULL;
    Formula * b = ❸ read_rpf();
    if (b==NULL) return NULL;
    return new Formula(‘C’, s, a, b); }
  else
    return NULL; }

somewhere else: Formula * f = ❶ read_rpf();
maybe main
```

---

14. Finally, after point 3, a new complex formula is created, and the function returns. This time we return right back to point 1 in main, to set the value of f.



And they all lived happily ever after.