Reminders from basic probability theory:

If the probability of reading any individual bit wrongly is called \( P \), then the probability of reading any individual bit correctly is \( 1 - P \), and the probability of reading a whole sequence of \( N \) bits correctly is \( (1-P)^N \).

Reminder from common sense:

For any storage medium that works even a little bit, \( P \) will be very small. Really small. But never zero.

Reminder from mathematics:

\[
(1-P)^N \text{ is evaluated as } 1 - NP \\
+ \frac{N(N-1)P^2}{2} \\
- \frac{N(N-1)(N-2)P^3}{6} \\
+ \frac{N(N-1)(N-2)(N-3)P^4}{24} \\
- \frac{N(N-1)(N-2)(N-3)(N-4)P^5}{120} \\
+ \frac{N(N-1)(N-2)(N-3)(N-4)(N-5)P^6}{720} \\
.....
\\
..... \text{ all the way up to} \\
+ P^N
\]

which takes a lot of calculating

Another reminder from probability:

If \( P \) is very small, then \( (1-P)^N \) is the same as \( e^{-PN} \) which is easy to work out.

Another reminder from mathematics:

\( e \) is the special magic number 2.7182818284590452353602874713527......

So...
Combined error rates are very easy to calculate for any realistic system.

Let’s say the single bit error rate is $10^{-5}$, meaning that if you attempt to read a single bit there is a 1 in 100,000 chance of getting it wrong. Or that if you experimentally read a bit 100,000 times under identical circumstances, you would expect to get it wrong once. Or that if you read 100,000 bits, you expect on average one error.

The chances of reading a whole block of $N$ bits successfully, i.e. without any errors will be

$$e^{-0.00001N}$$

<table>
<thead>
<tr>
<th>number of bits in a block</th>
<th>chances of reading it successfully</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>99%</td>
</tr>
<tr>
<td>4,096</td>
<td>96%</td>
</tr>
<tr>
<td>10,000</td>
<td>90%</td>
</tr>
<tr>
<td>20,000</td>
<td>82%</td>
</tr>
<tr>
<td>40,000</td>
<td>67%</td>
</tr>
<tr>
<td>100,000</td>
<td>37%</td>
</tr>
<tr>
<td>200,000</td>
<td>14%</td>
</tr>
<tr>
<td>400,000</td>
<td>1.8%</td>
</tr>
<tr>
<td>1,000,000</td>
<td>0.0045%</td>
</tr>
</tbody>
</table>

So you see why block size has to be kept relatively small.

A block on a hard disc has 4096 bits.
An error rate of $10^{-5}$ for a modern hard disc drive is not good.